An Efficient Parallel Algorithm for Numerical Solution of Low Dimension Dynamics Problems

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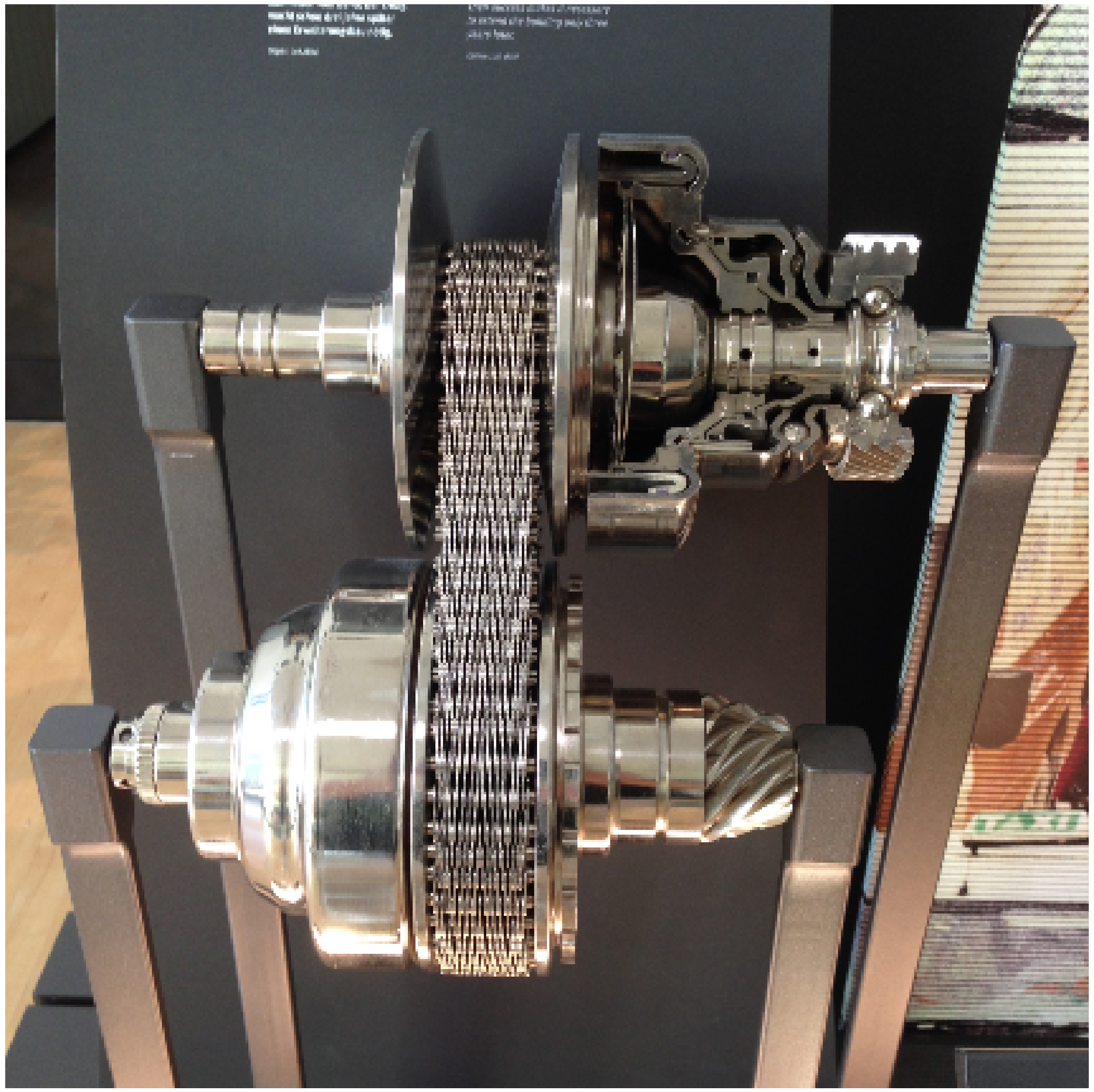
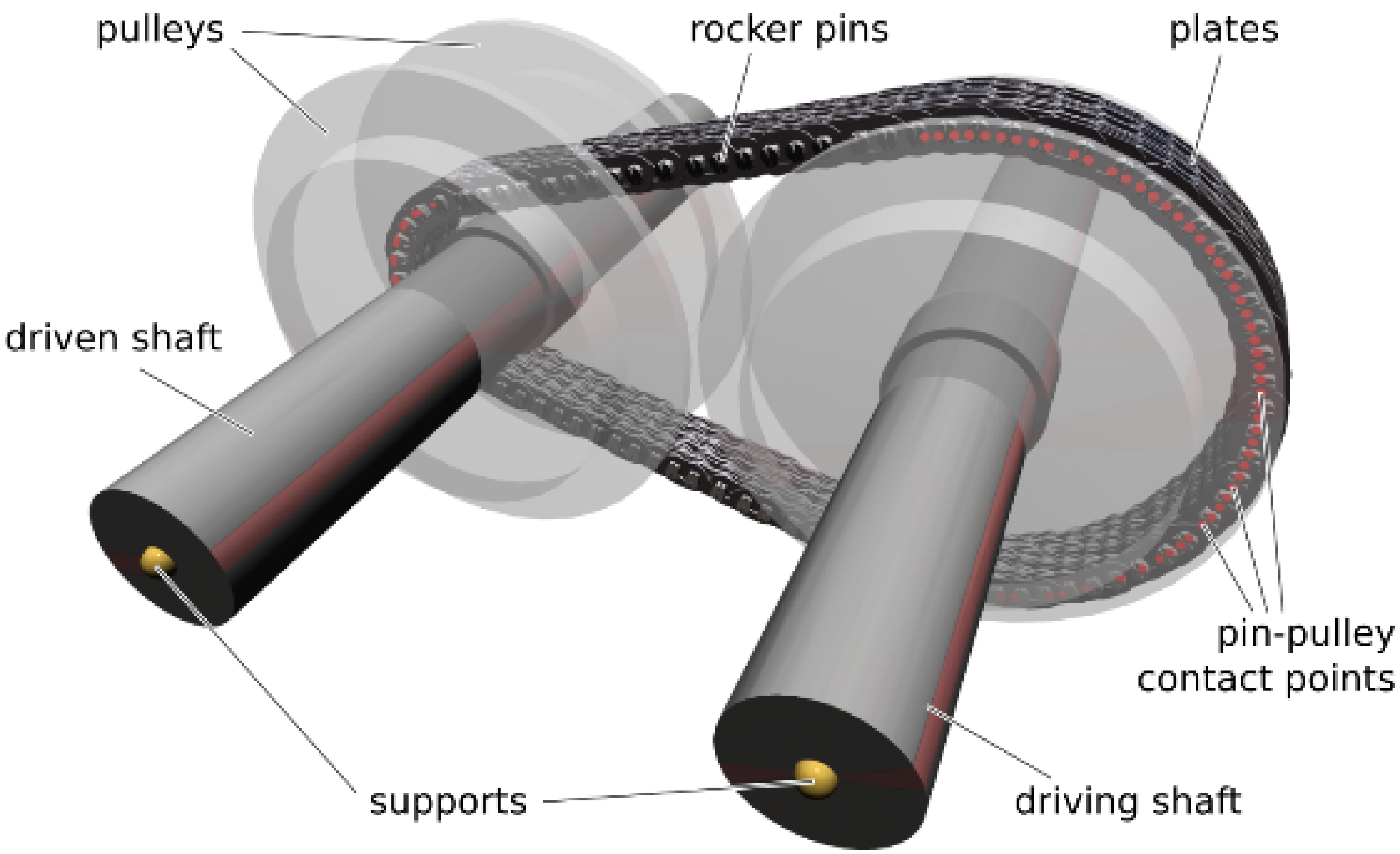
Model overview

Real device

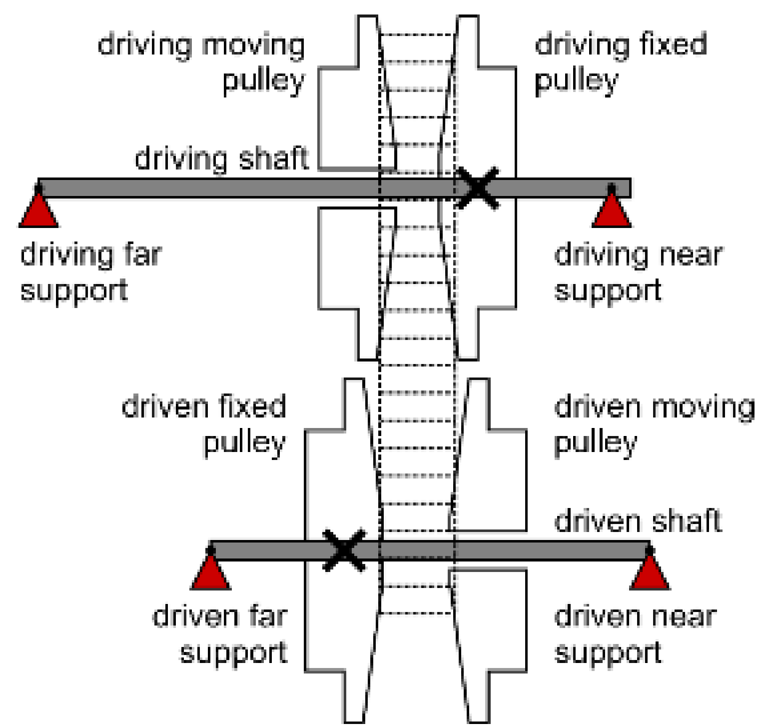


Model overview

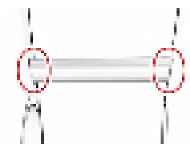
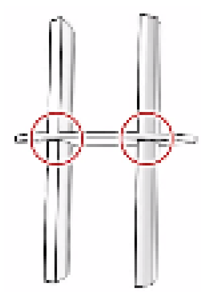
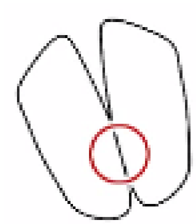
3D view

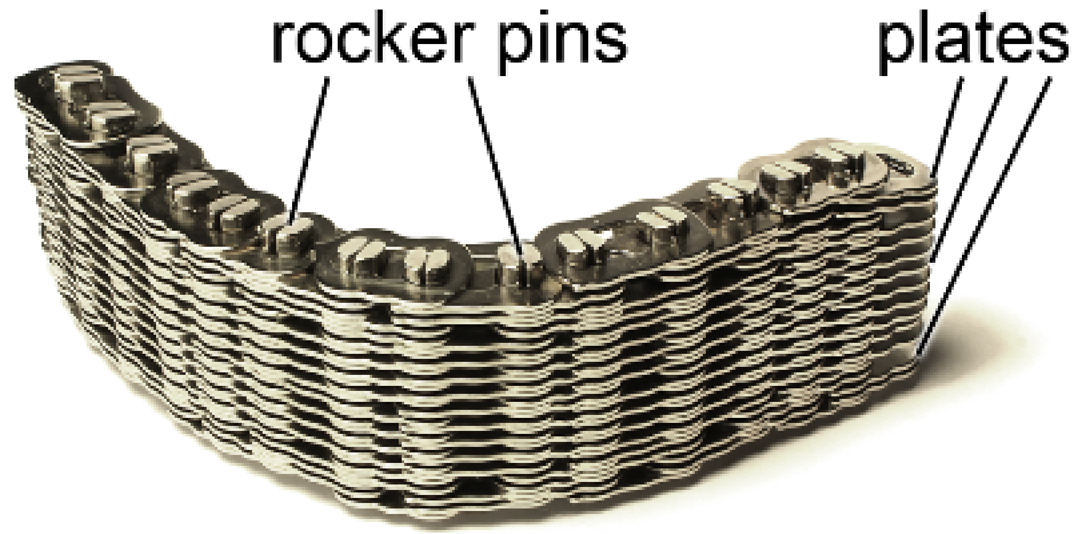


top view



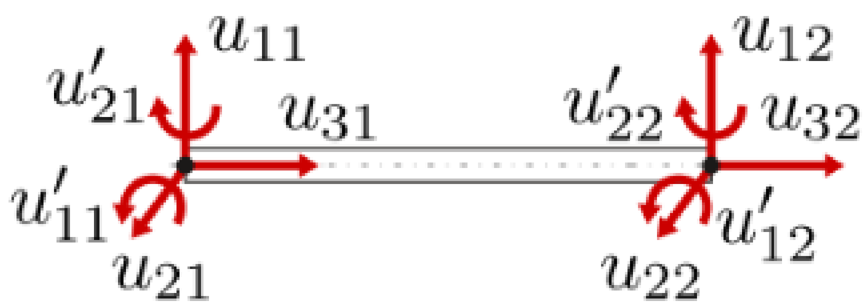
Model overview

- The chain consists of plates and rocker pins
- Each pin has two halves rolling over each other
- There are many contact interactions
  - pin — pulley 
  - pin — plate 
  - pin — pin 

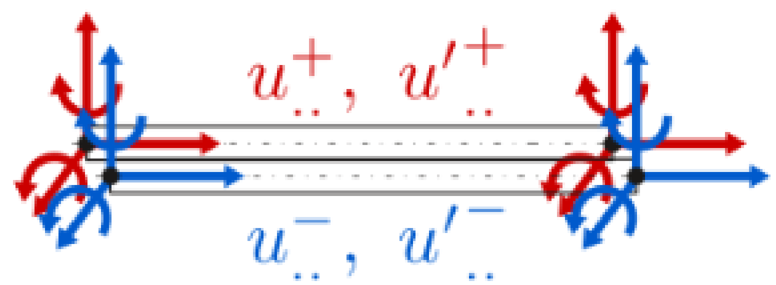


Model overview

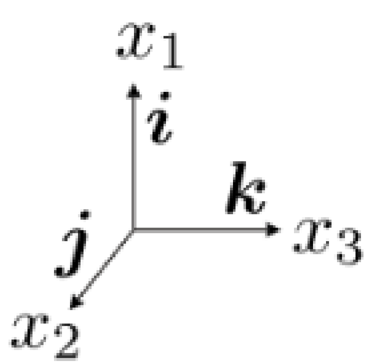
21 generalized coordinates per chain link



pin half



pin



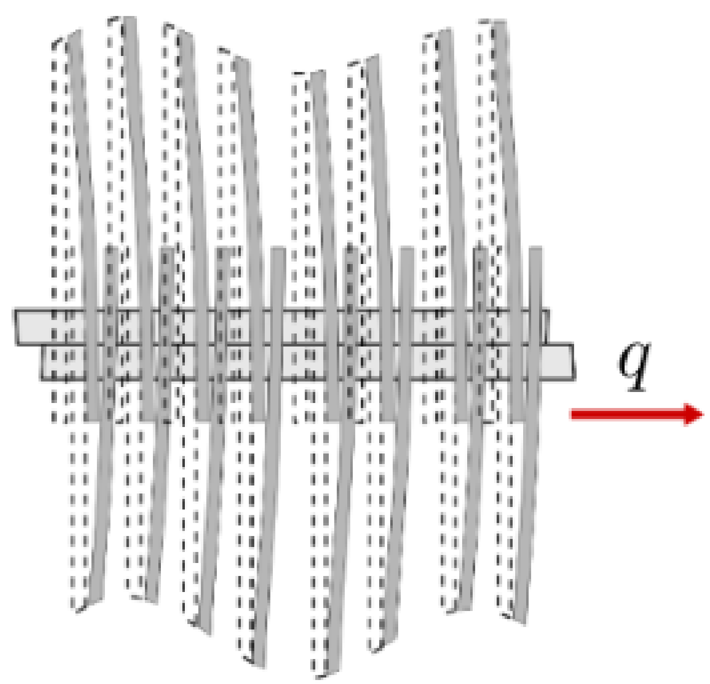


plate pack

Equations of motion

- Lagrange equations: $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \tilde{Q}$
- lead to $\mathbf{A}(q)\,\ddot{q} = F(t, q, \dot{q}\,)$
  - The inertia matrix $\mathbf{A}$ is sparse block-diagonal
  - Sometimes it really depends on q
- In the normal form, ODE system is $A\dot{x} = f(t, x)$
  - $\begin{matrix} q \equiv u \\ \dot{q} \equiv v \end{matrix}, \quad A = \begin{bmatrix} I & 0 \\ 0 & \mathbf{A} \end{bmatrix},$
  - $x = \begin{bmatrix} u \\ v \end{bmatrix}, \quad f = \begin{bmatrix} v \\ F(t, u, v) \end{bmatrix}$

Integration Scheme

- Solving IVP for

$$A\dot{x} = f(t,x) = [v, F(t,u,v)]^T \quad \Rightarrow \quad \dot{x} = \tilde{f}(t,x)$$

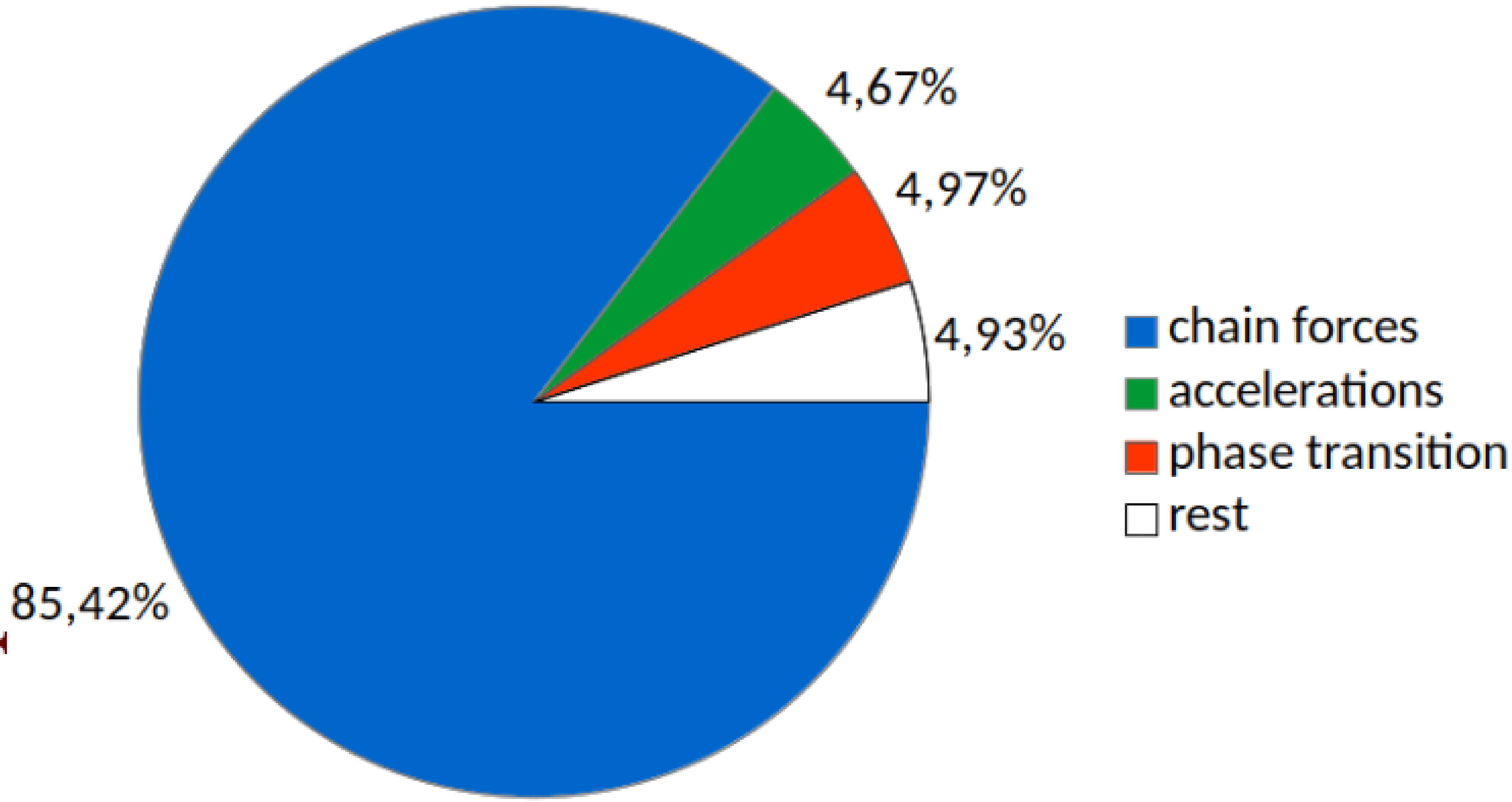
- Currently using explicit RK4 scheme

$$\begin{aligned}
k_1 &= \tilde{f}\left(t^{(n)}, x^{(n)}\right), \\
k_2 &= \tilde{f}\left(t^{(n)} + \tfrac{h}{2}, x^{(n)} + \tfrac{h}{2}k_1\right), \\
k_3 &= \tilde{f}\left(t^{(n)} + \tfrac{h}{2}, x^{(n)} + \tfrac{h}{2}k_2\right), \\
k_4 &= \tilde{f}\left(t^{(n)} + h, x^{(n)} + hk_3\right), \\
x^{(n+1)} &= x^{(n)} + \tfrac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right).
\end{aligned}$$

- Each evaluation of $\tilde{f} \Rightarrow$ one evaluation of $F(t,u,v)$ and Cholesky reverse pass for A
- Each integration step ends with phase transition procedure

Parallelization

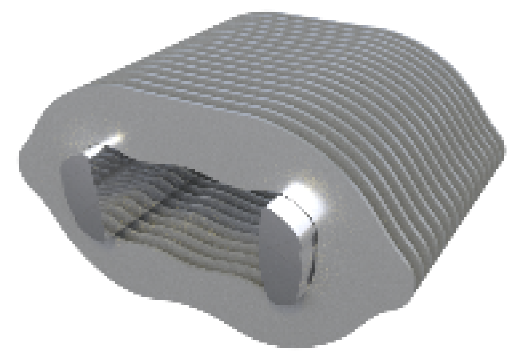
Time distribution in sequential code

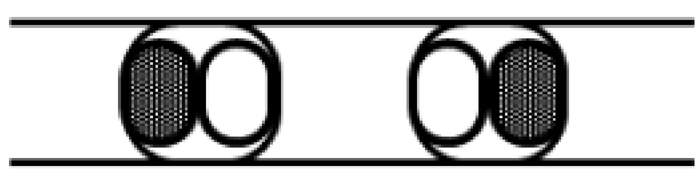


Parallelization

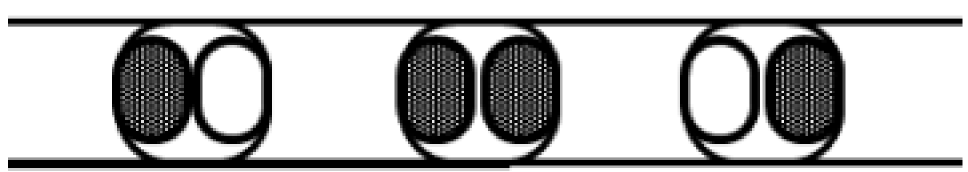
of chain forces calculation

Possible parallelization grains arise from forces kinds:





- Link forces
  - Forces due to link plates deformations

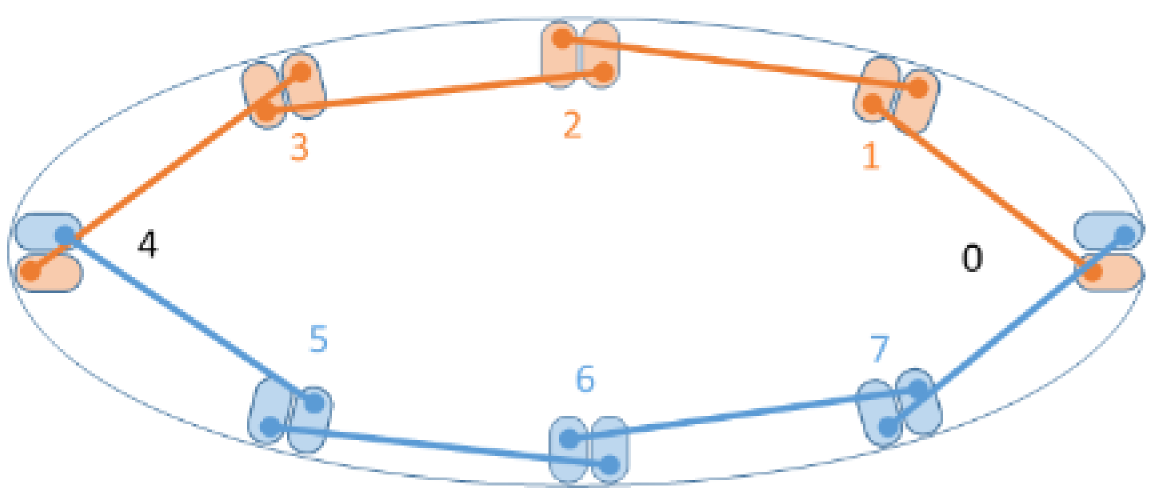


- Pin forces
  - Contact interactions between pin halves

Parallelization

of chain forces calculation

Workload distribution in the case of 2 threads



Thread 1	Thread 2
Independent calculation of pin/link forces for pins:	
1, 2, 3	5, 6, 7
Barrier	
Independent calculation of pin forces for:	
Pin 0	Pin 4

Simulation Results

CVT model with chain containing 84 pins considered.

The resulting system of ODEs consists of approx. 3600 equations.

$2 * 10^{-3}$ s of model time require 7min.22s of processor time in sequential mode.

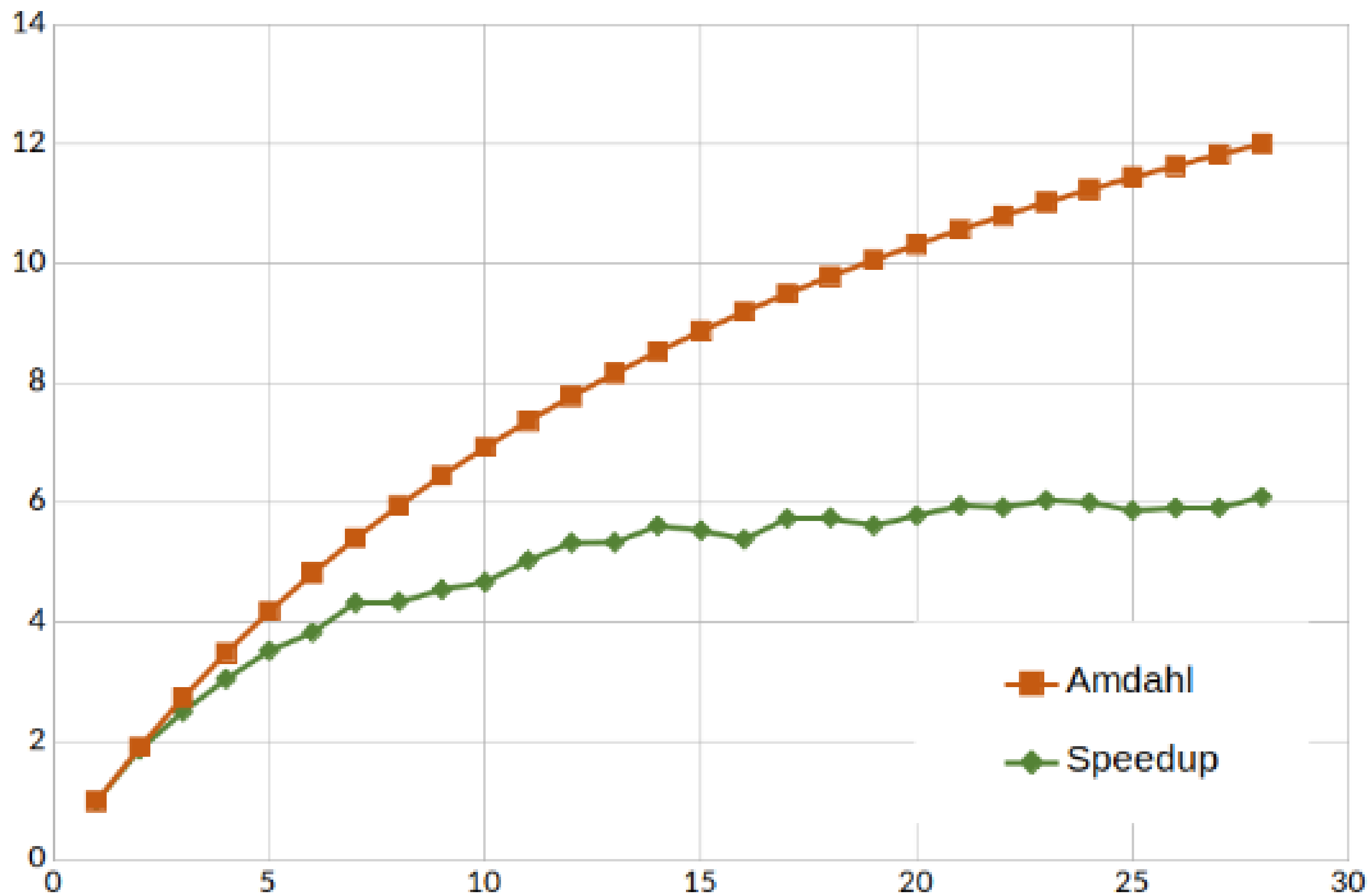
"Polytechnic RSK Tornado"

of Supercomputer Center Polytechnic of SPbPU

Cores per socket	14
NUMA Nodes	2
CPUs	Intel Xeon CPU E5-2697 v3 2.60GHz
Linux	CentOS Linux release 7.0.1406 (Core)
C++ compiler	Intel 2017.5.239

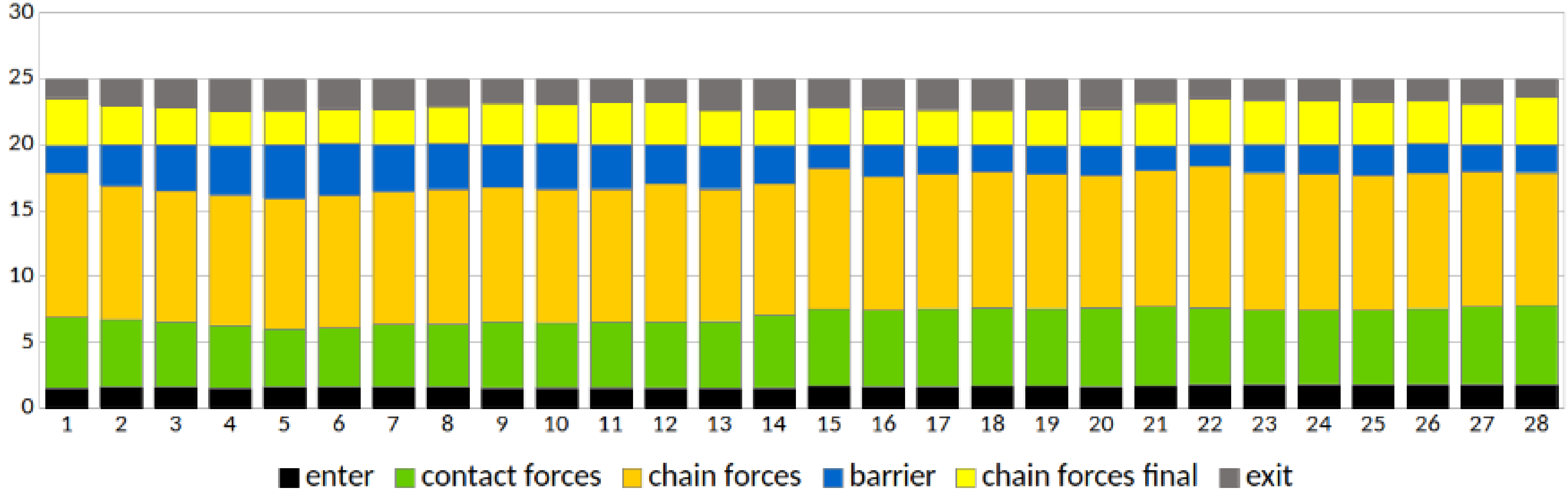
Parallelization

The speedup of the whole simulation as a function of thread count



Amdahl's law curve corresponds the fraction of serial work equal to 5%

Parallelization



- There is imbalance between threads in contact forces calculation
- Significant piece of time is spent in OpenMP code:

enter	7%
barrier	11%
exit	8%

Conclusions

- Maximal speedup of the whole simulation equal to 6 achieved for given hardware
- Significant scalability can be achieved only up to 10-12 threads. Further thread count increment leads to saturation.
- One of the sources of load imbalance is contact forces calculation.
- Significant overhead time presents at the start and the end of OpenMP section when the number of threads is large.

Thank you